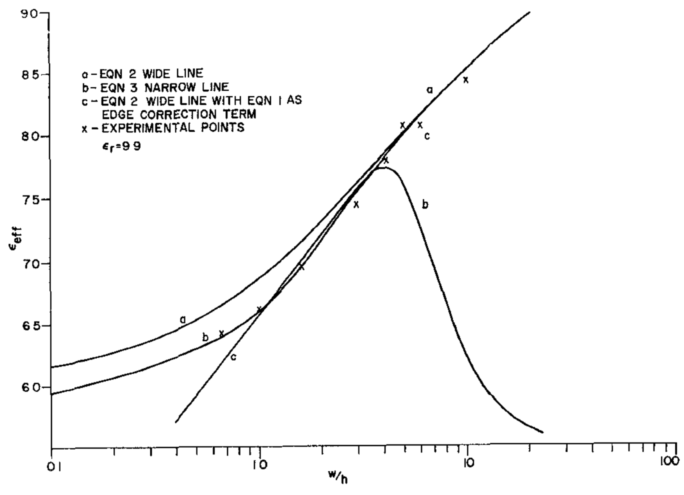

Fig. 3. Microstrip transmission-line effective dielectric constant as a function of the w/h ratio.

expression $(h \ln 4)/\pi$ to account for the edge correction for all values of w/h larger than unity, his reasoning being that the actual edge correction approaches this limiting value very rapidly as w/h approaches and exceeds unity. While attempting to define a crossover point between the narrow- and wide-line cases, the present authors considered the possibility that the edge-correction term for the wide-line case should be a function of the w/h ratio. It was empirically determined that if the edge-correction term in (2) was described in the same fashion as the open-end correction developed in this short paper (1), then a definite crossover point existed between the narrow- and wide-line approximations and the range of validity of the wide-line approximation was extended. Values of ϵ_{eff} given by substituting (1) into (2) are plotted in Fig. 3 (curve c). It is seen that curve c closely approximates the narrow-line curve (curve b) for all values of w/h greater than 1.0 and less than about 3.5 and is asymptotic to curve a for $w/h > 3.5$. In order to test the validity of curve c, ϵ_{eff} was measured using a resonant-ring technique for various w/h ratios [6], [7]. The measured results are shown in Fig. 3 and good agreement is exhibited with the modified expression for $w/h > 1$ (and with curve b for $w/h < 3.5$).

III. Conclusions

An empirically derived relation is reported which characterizes the open-end effects in microstrip transmission lines on alumina substrates. It is shown that the same empirical equation may be used to describe the edge-correction term in Wheeler's general expression for the characteristic impedance of a wide line. It is demonstrated that with this modification, the wide-line approximation is valid over a much larger range of w/h values and that a distinct crossover point ($w/h \approx 1$) exists between the narrow-line and wide-line approximations.

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Extension of Digital Automatic Method for Measuring the Permittivity of Thin Dielectric Films

M. A. RZEPECKA AND M. A. K. HAMID

Abstract—The permittivity of thin dielectric films can be measured with good accuracy by employing a method recently reported by the authors, whereby the microwave oscillator frequency is automatically locked to the resonant frequency of the test cavity perturbed by the sample, thus leading to a digital readout of the frequency. However, the method is satisfactory only when the frequency shift caused by the presence of the test sample does not exceed the frequency lock-in bandwidth. By employing a search oscillator, controlled by the second harmonic of the modulation signal provided for the frequency locking, this limitation is removed, thus extending the capability of the method to thicker films and/or larger permittivities.

Introduction

The permittivity of thin dielectric films can be measured with considerable accuracy by a digital automatic method recently reported by the authors [1]. The method utilizes the cavity perturbation technique in which the frequency of the microwave oscillator is locked to the resonant frequency of the test cavity in the absence and presence of the test film. These frequencies can be measured very accurately by a digital frequency counter. However, the ranges of film thickness and material permittivity in which the method can be employed are limited since they are determined by the lock-in bandwidth of the frequency control loop. This bandwidth is directly related to the Q factor of the test cavity, which in turn should be large enough to assure a high frequency-stabilization factor, thus permitting only a small deviation of the oscillator frequency from the resonant frequency of the cavity.

In many applications of this measurement method, as for instance in the continuous monitoring of moisture content of sheet materials, the range of the shift in resonance frequency exceeds the lock-in bandwidth of the control loop. A method for extending the measuring range is described in this short paper.

Principle of Operation

The complete circuit is shown in Fig. 1 where the additional parts over the circuit previously reported in [1] are shown by dashed lines. The principle of operation of the locking system is also described in [1]. At the output of the linear homodyne detector there exists a signal at the modulation frequency which is used for the frequency lock, as well as a signal at the second harmonic frequency. Basic signal analysis [1]–[3] shows that the second harmonic signal is described by the equation

$$E(2\omega_m) = -E_{sc} \mid T(\omega) \mid \sin \alpha \sum_{q=1}^{\infty} \frac{r^q p_q'}{q(q+1)} G_{q+1,2} \tag{1}$$

where

$$\alpha = \theta - \arg \Gamma$$

$$\mid T(\omega) \mid = \frac{T(\omega_0)}{1 + 4Q_L^2 \left(\dfrac{\omega - \omega_0}{\omega_0}\right)^2} \tag{2}$$

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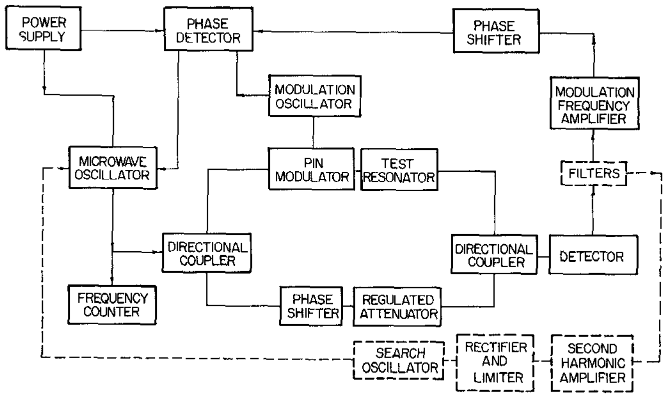

Fig. 1. Block diagram of the resonant frequency shift measurement setup with a search oscillator.

TABLE I

q	1	2	3	4	5
$G_{q,2}$	0	$m^2/2$	$3m^2/2$	$m^2\left(3+\frac{m^2}{2}\right)$	$5m^2\left(1+\frac{m^2}{2}\right)$

$$\arg \Gamma = \arctan\left(2Q_L \frac{\omega-\omega_0}{\omega_0}\right) \tag{3}$$

and

E_{sc} amplitude of subcarrier signal,
$T(\omega_0)$ transmission coefficient of cavity at resonance,
ω_0 cavity resonant frequency in radians,
ω microwave oscillator frequency in radians,
Q_L loaded Q factor of cavity,
θ $=180-\phi$; ϕ is the angle between carrier and subcarrier signals,
r $=E_{sc}/E_c$,
E_c amplitude of carrier signal,

$$P_q' = \frac{dP_q\left\{\cos\left[\theta-\arctan 2Q_L\left(\frac{\omega-\omega_0}{\omega_0}\right)\right]\right\}}{d\left[\theta-\arctan 2Q_L\left(\frac{\omega-\omega_0}{\omega_0}\right)\right]},$$

$P_q(x)$ Legendre polynomials of order q and argument x,
$G_{q,2}$ special functions of modulation index m tabulated in [3] and listed for the first five terms in Table I.

Since in practice $r \leq 0.1$, all the terms in the sum beyond the first may be neglected. Thus (1) reduces to

$$E(2\omega_m) = -r\frac{E_{sc}}{4}\,|T(\omega)|\,m^2\sin^2\alpha. \tag{4}$$

In order to assure frequency lock by the signal at the first harmonic, the angle ϕ between the carrier and subcarrier signals must have specific values which depend on r and which are listed in [1]. The proper angle for the frequency lock is denoted by ϕ_0. The change in the angle $(\theta-\arg\Gamma)$ as a function of frequency, resulting from variations of $\arg\Gamma$ with frequency near ω_0, is shown in Fig. 2 for a cavity coupling coefficient (β) smaller than unity. The frequencies ω_1, ω', and ω'' denoted in Fig. 2 are given by

$$\omega_1 = \omega_0\left[1+\frac{\frac{\pi}{2}-\phi_0}{2Q_L}\right] \tag{5}$$

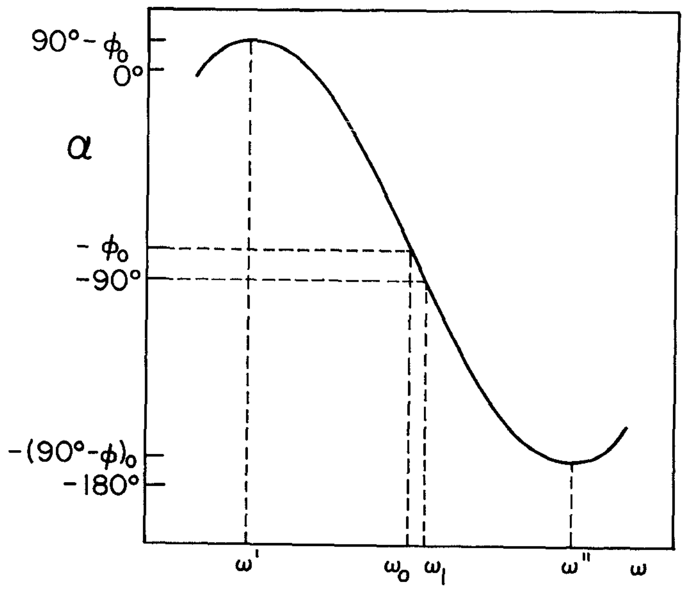

Fig 2. Changes of the angle α versus frequency for $\phi=\phi_0$.

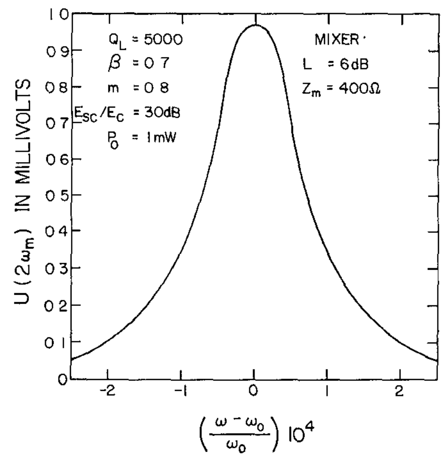

Fig. 3. Amplitude of the signal of the second harmonic of the modulation frequency.

$$\omega' = \omega_0\left[1-\frac{1}{2Q_L}\left(\frac{1-\beta}{1+\beta}\right)^{1/2}\right] \tag{6}$$

$$\omega'' = \omega_0\left[1+\frac{1}{2Q_L}\left(\frac{1-\beta}{1+\beta}\right)^{1/2}\right]. \tag{7}$$

Thus the second harmonic signal described by (4) is maximum at ω_1, which in practice is slightly different from the resonant frequency, and drops to zero outside the resonant cavity bandwidth. This signal is shown in Fig. 3 after detection for typical values of $T(\omega_0)$, Q_L, β, r, and a typical mixer-detector circuit. This signal may be utilized for the control of a search oscillator. This oscillator provides a swept voltage signal to the voltage-controlled electrode of the microwave oscillator as long as there is no signal at its control terminals. If the second harmonic signal appears, then it should switch off the search oscillator, thus permitting operation analogous to a microwave frequency synchronizer. By these means, the microwave oscillator frequency is shifted close enough to the cavity resonant frequency that it lies in the band of frequency locking.

This technique was checked for an X-band reflex klystron oscillator, and satisfactory performance was achieved in the band corresponding to about 80 percent of the klystron mode of operation bandwidth.

Conclusions

The utilization of the second harmonic of the amplitude-modulated signal for the control of the search oscillator enables shifting the microwave oscillator frequency close enough to the cavity resonant frequency for the automatic frequency locking circuit to start the control. This makes it possible to increase the range of thickness and permittivity of the dielectric films measured by the system.

Acknowledgment

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Anomalous Convergence of Iterative Methods in the Numerical Solution of Electromagnetic Problems

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***Abstract*—Iterative methods applied to eigenvalue equations can lead to anomalous convergence. It is shown that this can occur when some of the eigenvalues are complex and the corresponding eigenvectors satisfy a particular condition. A method of distinguishing between anomalous and effective convergence is indicated.**

Many electromagnetic problems can be solved by the finite difference technique, which leads to a system of difference equations whose coefficient matrix A is generally real. The approximate solution of the continuous problem is then obtained by solving the matrix eigenvalue problem $(A-\lambda I)x=0$. If the matrix A is very large it is necessary to use an iterative method for the computation of the eigenvalues.

A useful procedure [1] that allows the computation of all the real eigenvalues of A is to introduce the semidefinite positive matrix

$$C(\lambda) = (A-\lambda I)^T(A-\lambda I) \tag{1}$$

and then to compute an eigenvalue λ in the following manner.

1) Equation $C(\lambda^{(0)})x=0$ ($\lambda^{(0)}$ being a guess at λ) is solved by an iterative method, e.g., successive displacements, starting with an arbitrary real vector $x^{(0)}$.

2) A reestimate $\lambda^{(1)}$ is computed from the Rayleigh quotient and the value obtained replaces $\lambda^{(0)}$ in C.

3) An alternation between steps 1 and 2 is carried out until $\lambda^{(i)}$ seems to have converged to λ.

The described procedure generally works satisfactorily; however, we verified that anomalous convergences can occur when the matrix A is unsymmetric. It is the purpose of this short paper to discuss these anomalies and to show how to identify them.

Let us refer to a very simple example. Consider the 5×5 circulant matrix [2] whose first row is

$$a_{1j} = [0, 1, 2, 3, 6] \tag{2}$$

and use it as a test matrix.

By using as iterative method in step 1 the method of successive displacements and continuing the procedure for calculating λ until $\lambda^{(i)}$ and $\lambda^{(i+1)}$ differ by less than 0.1 percent, we obtained the following results.

$\lambda^{(0)}$	$x^{(0)}$	(Convergence Value) λ_{conv}
−3	1, 2, 3, 4, 5	−4.115
3	1, 2, 1, 2, 1	−4.115
5	1, 2, 1, 2, 1	12

It is easy to verify that only $\lambda_{\text{conv}}=12$ is an eigenvalue of (2), while $\lambda_{\text{conv}}=-4.115$ is an anomalous convergence value. However, it can be noticed that the value -4.115 corresponds to the real part of the complex eigenvalues of A. The behavior shown in the example has also been found in solving other real matrices obtained from electromagnetic problems using the finite difference technique. The anomalous convergence values are always found to be the real part of complex eigenvalues. The above result must, therefore, be borne in mind in the case of problems that are not schematized by a symmetric matrix, since, as complex eigenvalues may exist, there is the possibility that the iterative procedure may give rise to a convergence towards a quantity that does not correspond to an actual eigenvalue. We will, therefore, examine in detail the whole procedure for studying in which cases an anomalous convergence may occur.

Let $\mathcal{L}(C, \lambda^{(0)})$ be the error-reducing (or iteration) matrix [2] relative to the iterative method adopted to solve $C(\lambda)x=0$, and let μ_i and I_i be the eigenvalues and the corresponding eigenvectors of $\mathcal{L}$. Both μ_i and I_i are in general complex. Supposing that the matrix A is not defective [3], the initial real vector $x^{(0)}$ can be expressed as

$$x^{(0)} = \sum_{i=1}^{n} (\alpha_i I_{ir} + \beta_i I_{ij}) \tag{3}$$

where α_i and β_i are real constants and $I_i = I_{ir} + jI_{ij}$. Performing s iterations we obtain

$$x^{(s)} = \sum_{i=1}^{n} (\alpha_i \mathcal{L}^s I_{ir} + \beta_i \mathcal{L}^s I_{ij}) \tag{4}$$

where a generic eigenvalue and the corresponding eigenvector are related by

$$\begin{aligned} \mathcal{L}^s I_r &= |\mu|^s (I_r \cos s\theta - I_j \sin s\theta) \\ \mathcal{L}^s I_j &= |\mu|^s (I_j \cos s\theta + I_r \sin s\theta) \end{aligned} \tag{5}$$

with $\mu = |\mu| e^{j\theta}$.

Let μ_1 be the eigenvalue having the greatest absolute value. Let us distinguish the two cases

1) μ_1 real ($I_{1r} = I_1$; $I_{1j} = 0$).

Equation (4) for s increasing tends to

$$x^{(s)} = \alpha_1 \mu_1^s I_1.$$

Therefore, $x^{(s)}$ tends to assume the form of I_1 and the Rayleigh quotient tends to become constant. Moreover, the Rayleigh quotient gives a value $\lambda^{(i+1)}$ of λ that is closer than $\lambda^{(i)}$ to a true eigenvalue.

2) μ_1 complex.

Since the matrix $\mathcal{L}$ is real, $\mu_2 = \mu_1^*$ and $I_2 = I_1^*$. Taking account of (5), it is found that, as s increases, $x^{(s)}$ tends to

$$x^{(s)} = (\alpha_1 + \alpha_2)\mathcal{L}^s I_{1r} + (\beta_1 - \beta_2)\mathcal{L}^s I_{1j} = a(s) I_{1r} + b(s) I_{1j} \tag{6}$$

where $a(s)$ and $b(s)$ are oscillating functions of s. Consequently, $x^{(s)}$ oscillates in sign as s varies. However, let us consider the Rayleigh quotient

$$\frac{x^{(s)T} A x^{(s)}}{x^{(s)T} x^{(s)}} = \frac{(aI_{1r}^T + bI_{1j}^T) A (aI_{1r} + bI_{1j})}{(aI_{1r}^T + bI_{1j}^T)(aI_{1r} + bI_{1j})}.$$

The quotient, which will generally vary with a and b (i.e., with s),

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